Engineering Electromagnetics

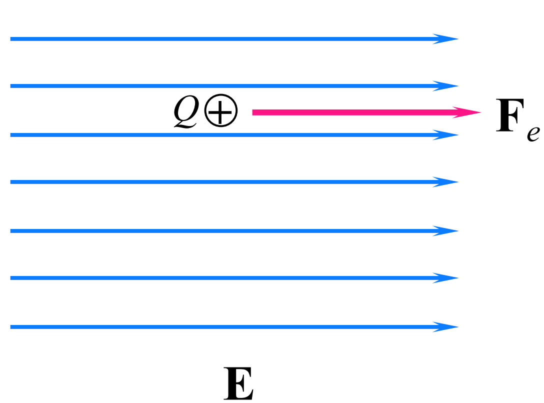
Chapter 8:

Magnetic Forces, Torque, and Inductance

Force on a Moving Charge

The forces exerted on a point charge by electric and magnetic fields are:

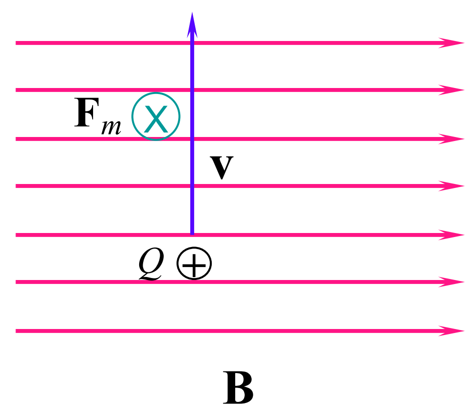
charge can be stationary
or moving



$$\mathbf{F}_e = Q\mathbf{E}$$

in the direction of **E**

charge is moving
at velocity **v**

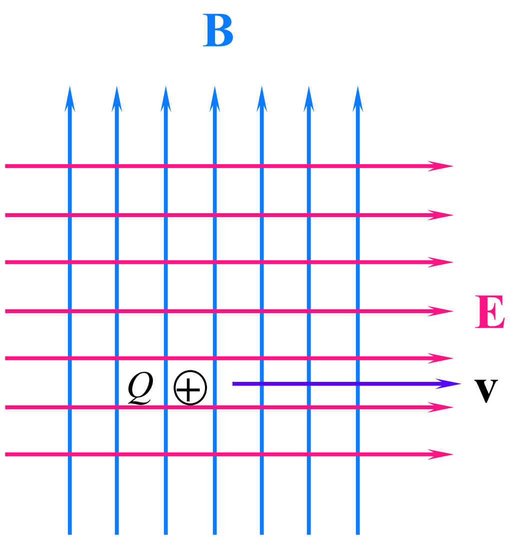


$$\mathbf{F}_m = Q\,(\mathbf{v} \times \mathbf{B})$$

into the screen

Lorentz Force Law

Generally, with both electric and magnetic fields present, we have both forces:



The electric field will, in this case, accelerate the charge in the direction of **E**, making it cross the **B** field lines in the perpendicular sense; this gives a magnetic force component that is out of the screen

The total force on the moving charge is then the sum of the two, or

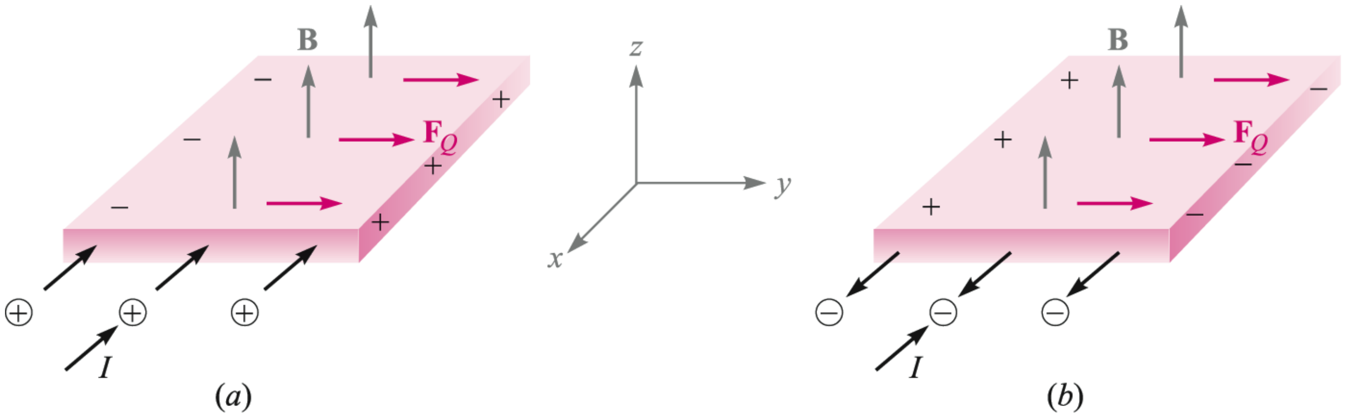
$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

This is the *Lorentz Force Law* (sometimes called the "fifth Maxwell equation")

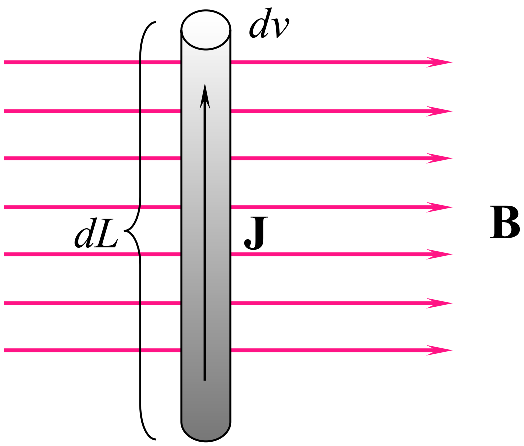
Hall Effect

When a **B** field is applied in a direction perpendicular to a current, **positive and negative carriers will be displaced slightly**, as shown, as a result of the magnetic forces on the moving charges.

This produces a measurable voltage, known as the *Hall Voltage*.



Force on a Differential Current Element



Consider a small segment (length dL) of current in the form of a volume current density $\mathbf{J}$, suspended in a magnetic field, $\mathbf{B}$. The current element has volume dv.

We know that current density is volume charge density moving at velocity $\mathbf{v}$:

$$\mathbf{J} = \rho_\nu \mathbf{v}$$

..and we can write the differential force on a differential charge, dQ:

$$d\mathbf{F} = dQ\,\mathbf{v} \times \mathbf{B} \quad \text{where} \quad dQ = \rho_\nu d\nu$$

Therefore:

$$d\mathbf{F} = \rho_\nu d\nu\,\mathbf{v} \times \mathbf{B}$$

..so that finally:

$$d\mathbf{F} = \mathbf{J} \times \mathbf{B}\,d\nu$$

Other Expressions for Differential Force

For volume current, surface current, or filament current, we have the appropriate expressions for differential current:

$$\mathbf{J}\,dv = \mathbf{K}\,dS = I\,d\mathbf{L}$$

The corresponding expressions for differential force within magnetic field **B** are:

$$d\mathbf{F} = \mathbf{J} \times \mathbf{B}\,dv$$ volume current density (three dimensions)

$$d\mathbf{F} = \mathbf{K} \times \mathbf{B}\,dS$$ surface current density (two dimensions)

$$d\mathbf{F} = I\,d\mathbf{L} \times \mathbf{B}$$ filament current of length dL (one dimension)

Evaluating the Total Force

In three or two dimensions, the net force is found by integrating over the volume or surface that the current occupies

$$\mathbf{F} = \int_{\text{vol}} \mathbf{J} \times \mathbf{B}\, dv$$

$$\mathbf{F} = \int_{S} \mathbf{K} \times \mathbf{B}\, dS$$

For a filament current, the total force will be the integral of the differential force, generally taken over the closed path that comprises the current:

$$\mathbf{F} = \oint I\, d\mathbf{L} \times \mathbf{B} = -I \oint \mathbf{B} \times d\mathbf{L}$$

For a straight filment of length *L*, having uniform current, and within a uniform field, this becomes:

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

which in turn reduces to $F = BIL \sin\theta$

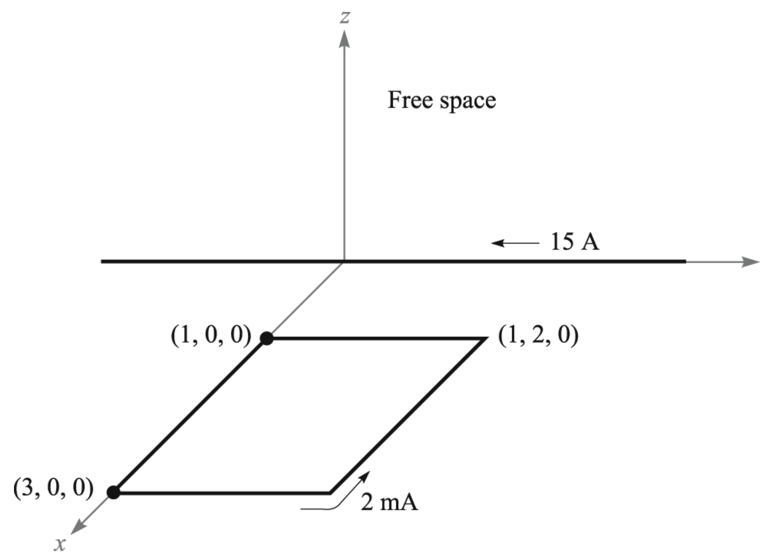
Example: Force on a Square Current Loop

The magnetic field arising from the straight wire, evaluated in the plane of the loop, is:

$$\mathbf{H} = \frac{I}{2\pi x}\mathbf{a}_z = \frac{15}{2\pi x}\mathbf{a}_z \text{ A/m}$$

The **B** field is then:

$$\mathbf{B} = \mu_0 \mathbf{H} = 4\pi \times 10^{-7}\mathbf{H} = \frac{3 \times 10^{-6}}{x}\mathbf{a}_z \text{ T}$$



The total force on the loop is then found by evaluating:

$$\mathbf{F} = -I \oint \mathbf{B} \times d\mathbf{L}$$

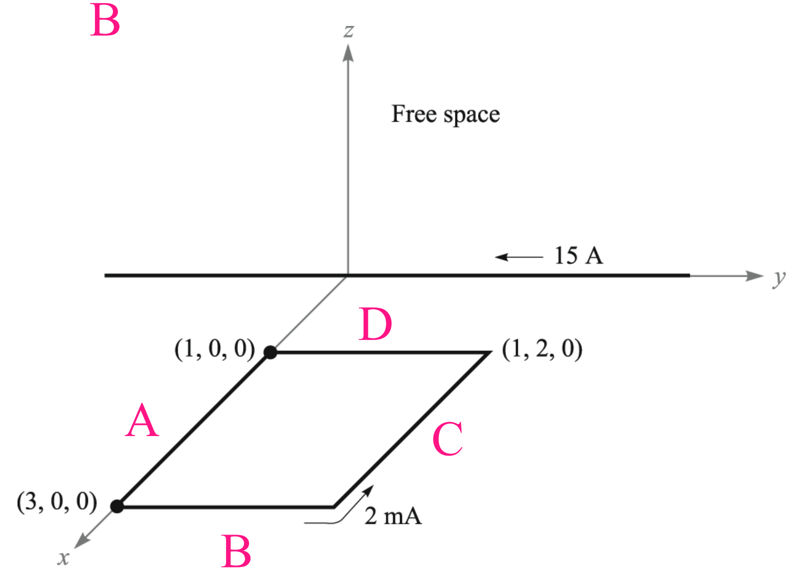
over all four loop segments

Example: Continued

The integral becomes:

$$\mathbf{F} = -2 \times 10^{-3} \times 3 \times 10^{-6} \Bigg[\underbrace{\int_{x=1}^{3} \frac{\mathbf{a}_z}{x} \times dx\, \mathbf{a}_x}_{\text{A}} + \underbrace{\int_{y=0}^{2} \frac{\mathbf{a}_z}{3} \times dy\, \mathbf{a}_y}_{\text{B}}$$

$$+ \underbrace{\int_{x=3}^{1} \frac{\mathbf{a}_z}{x} \times dx\, \mathbf{a}_x}_{\text{C}} + \underbrace{\int_{y=2}^{0} \frac{\mathbf{a}_z}{1} \times dy\, \mathbf{a}_y}_{\text{D}} \Bigg]$$



$$= -6 \times 10^{-9} \left[\ln x \Big|_1^3 \mathbf{a}_y + \frac{1}{3} y \Big|_0^2 (-\mathbf{a}_x) + \ln x \Big|_3^1 \mathbf{a}_y + y \Big|_2^0 (-\mathbf{a}_x) \right]$$

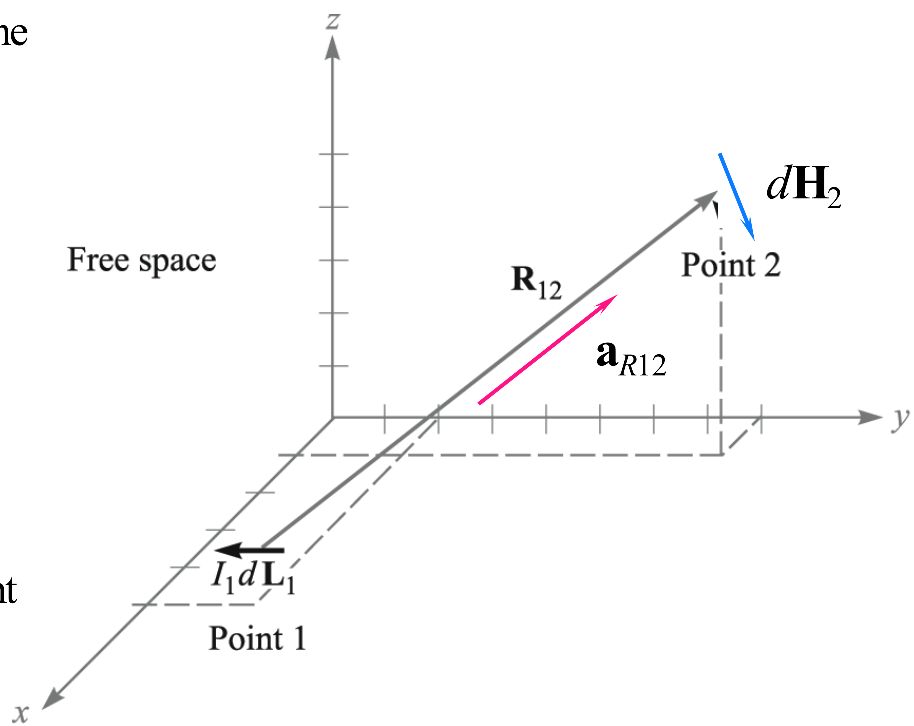
$$= -6 \times 10^{-9} \left[(\ln 3)\mathbf{a}_y - \frac{2}{3}\mathbf{a}_x + \left(\ln \frac{1}{3} \right) \mathbf{a}_y + 2\mathbf{a}_x \right]$$

$$= \underline{-8\mathbf{a}_x \text{ nN}}$$

Force Between Differential Current Elements

We use the Biot-Savart Law to find the differential magnetic field at Point 2 that arises from the differential current element at Point 1:

$$d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}$$



The flux density associated with this field will generate a force on an additional current element at Point 2

Force Between Differential Current Elements

A second differential current element is now placed at Point 2. It will experience a force given by:

$$d(d\mathbf{F}_2) = I_2 d\mathbf{L}_2 \times d\mathbf{B}_2$$

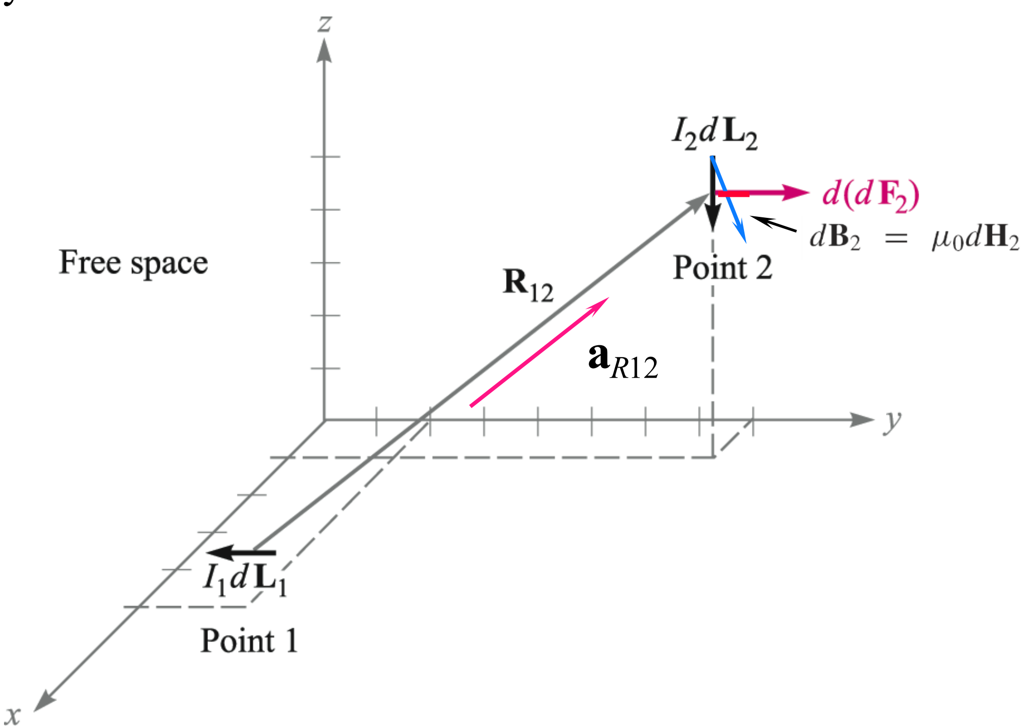
Then with:

$$d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}$$

where: $d\mathbf{B}_2 = \mu_0 d\mathbf{H}_2$

we finally obtain:

$$d(d\mathbf{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi R_{12}^2} d\mathbf{L}_2 \times (d\mathbf{L}_1 \times \mathbf{a}_{R12})$$

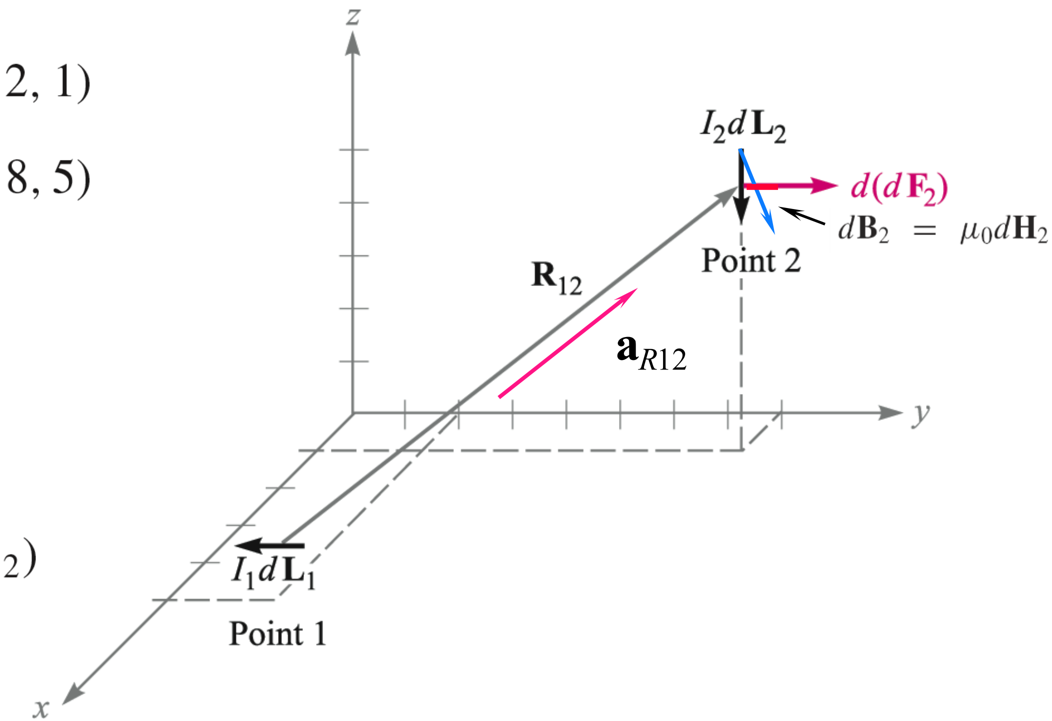


Example (8.2)

Given: $I_1 d\mathbf{L}_1 = -3\mathbf{a}_y \text{A}\cdot\text{m at } P_1(5, 2, 1)$

$I_2 d\mathbf{L}_2 = -4\mathbf{a}_z \text{A}\cdot\text{m at } P_2(1, 8, 5)$

Then $\mathbf{R}_{12} = -4\mathbf{a}_x + 6\mathbf{a}_y + 4\mathbf{a}_z$



Substitute these into:

$$d(d\mathbf{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi R_{12}^2} d\mathbf{L}_2 \times (d\mathbf{L}_1 \times \mathbf{a}_{R12})$$

Obtain:

$$d(d\mathbf{F}_2) = \frac{4\pi 10^{-7}}{4\pi} \frac{(-4\mathbf{a}_z) \times [(-3\mathbf{a}_y) \times (-4\mathbf{a}_x + 6\mathbf{a}_y + 4\mathbf{a}_z)]}{(16 + 36 + 16)^{1.5}}$$

$$= \underline{8.56\mathbf{a}_y \text{ nN}}$$

Force Between Current Filaments of Finite (or Infinite) Lengths

The force is found by integrating the differential filament result:

$$d(d\mathbf{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi R_{12}^2} d\mathbf{L}_2 \times (d\mathbf{L}_1 \times \mathbf{a}_{R12})$$

in which the integral is taken over the lengths of both filaments. To be complete, both integrals would be taken over the closed loops that the currents must form, thus the general expression:

$$\mathbf{F}_2 = \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[d\mathbf{L}_2 \times \oint \frac{d\mathbf{L}_1 \times \mathbf{a}_{R12}}{R_{12}^2} \right]$$

$$= \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[\oint \frac{\mathbf{a}_{R12} \times d\mathbf{L}_1}{R_{12}^2} \right] \times d\mathbf{L}_2$$

These integrals can be modified to incorporate specific limits, in order to find the force from a specified segment of a wire that acts on a specific segment of another wire

Example: Force Between Parallel Wires (the easy way)

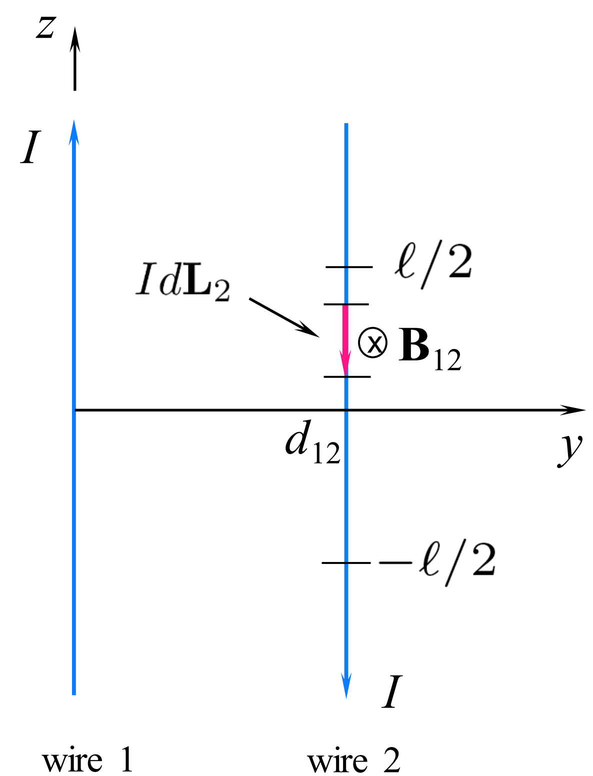
Consider the two wires shown here, carrying equal and opposite currents, I, and spaced by distance d_{12} along the y axis. The wires are oriented in the z direction and both are infinitely long.

The force is to be found on a length ℓ of wire 2, of equal extent above and below the y axis.

We can solve this one fairly quickly by observing that the **B** field from wire 1 at the location of wire 2 will be:

$$\mathbf{B}_{12} = -\frac{\mu_0 I}{2\pi d_{12}}\,\mathbf{a}_x$$

and $$I\,d\mathbf{L}_2 = -I\,dz_2\,\mathbf{a}_z$$

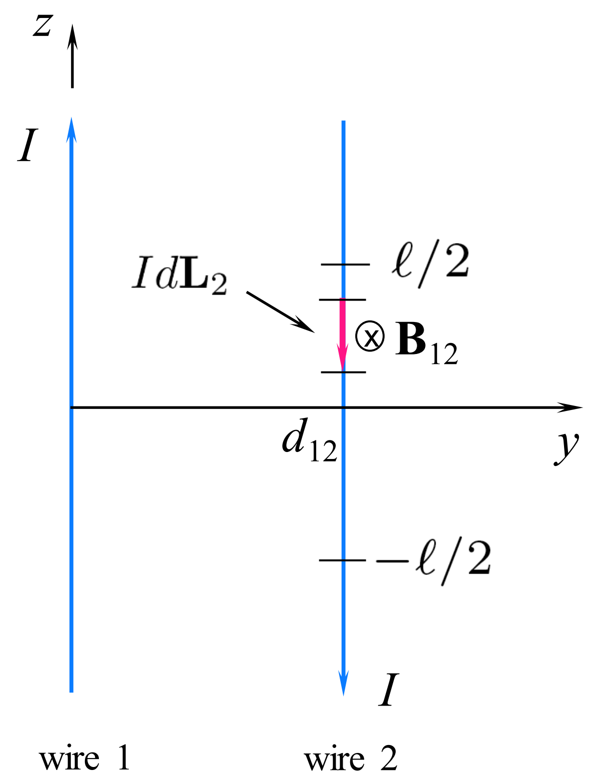


Example (continued)

We have: $$\mathbf{B}_{12} = -\frac{\mu_0 I}{2\pi d_{12}}\,\mathbf{a}_x \quad \text{and} \quad I d\mathbf{L}_2 = -I\,dz_2\,\mathbf{a}_z$$

Then the force acting on the differential element shown is: $$d\mathbf{F}_2 = I d\mathbf{L}_2 \times \mathbf{B}_{12}$$

$$= -I\,dz_2\,\mathbf{a}_z \times \left(-\frac{\mu_0 I}{2\pi d_{12}}\,\mathbf{a}_x\right) = \frac{\mu_0 I^2 dz_2}{2\pi d_{12}}\,\mathbf{a}_y$$

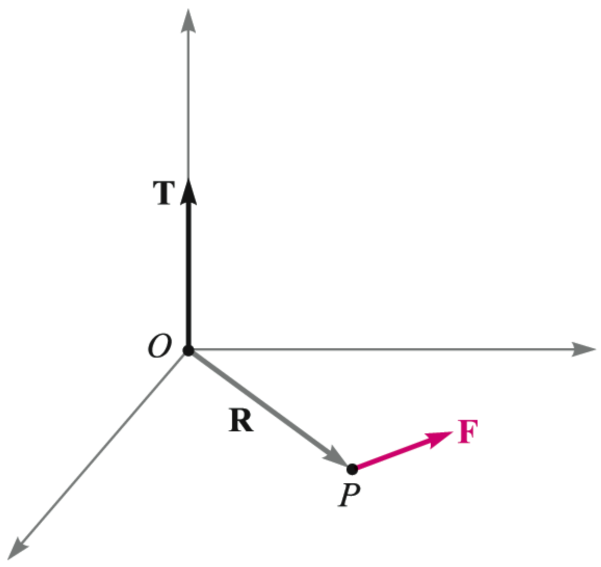


The total force on the length is therefore:

$$\mathbf{F}_2 = \int_{-\ell/2}^{\ell/2} I d\mathbf{L}_2 \times \mathbf{B}_{12} = \frac{\mu_0 I^2 \ell}{2\pi d_{12}}\,\mathbf{a}_y$$

….which is a repulsive force

Torque: Basic Definition



Given a force **F** at point *P*, the torque about the origin is a vector that is perpendicular to the plane containing **F** and position vector **R**. The torque vector is the cross product:

$$\mathbf{T} = \mathbf{R} \times \mathbf{F}$$

Torque: General Equation

Now, consider two *equal and opposite* forces, applied at points P_1 and P_2 as shown.

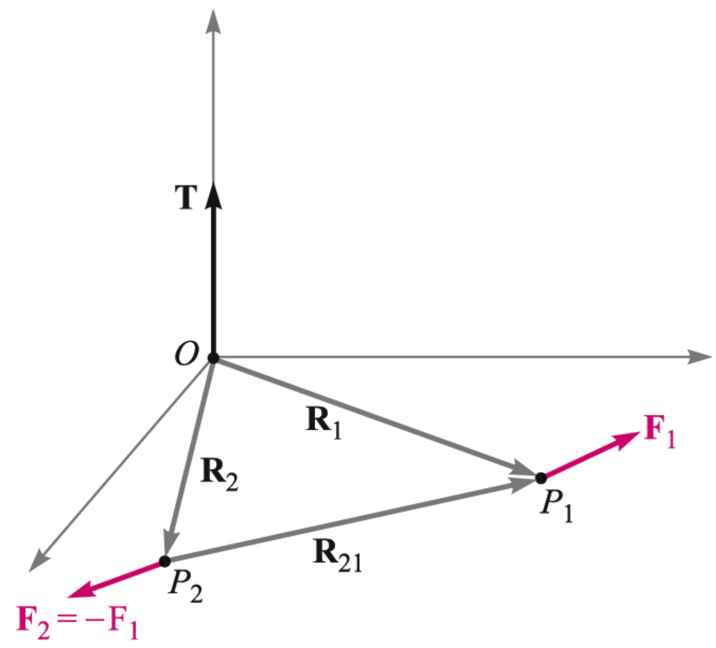
As the net force is zero, there is no translation of the object.

The net torque on the system will be:

$$\mathbf{T} = \mathbf{R}_1 \times \mathbf{F}_1 + \mathbf{R}_2 \times \mathbf{F}_2$$

But since $\mathbf{F}_2 = -\mathbf{F}_1$:

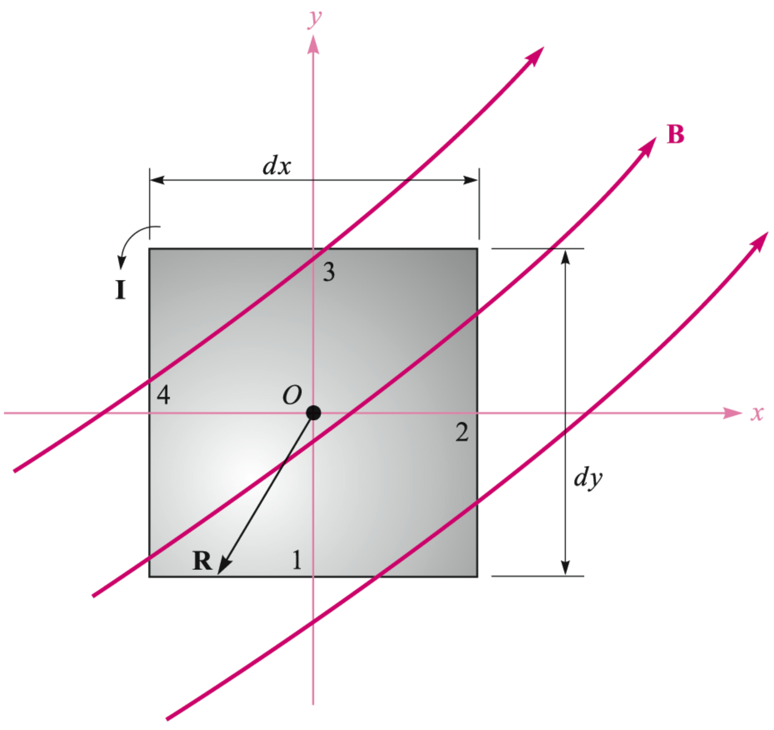
$$\mathbf{T} = (\mathbf{R}_1 - \mathbf{R}_2) \times \mathbf{F}_1 = \underline{\mathbf{R}_{21} \times \mathbf{F}_1}$$



This means that the *torque is independent of the choice of origin, provide the total force acting is zero*. This applies to any number of forces. And the origin can be located anywhere that is convenient.

Torque on a Differential Current Loop

The filament loop shown here lies in the *xy* plane with its center at the origin. Magnetic flux density **B** exists everywhere, and in a general direction. As the loop is of differential size, the magnitude of **B** is assumed uniform over the loop area, and has value B_0. Current I circulates around the loop.



The differential force acting on side 1 is:

$$d\mathbf{F}_1 = I\,dx\,\mathbf{a}_x \times \mathbf{B}_0$$

$$= I\,dx(B_{0y}\mathbf{a}_z - B_{0z}\mathbf{a}_y)$$

Torque on a Differential Current Loop

We have the differential force acting on side 1:

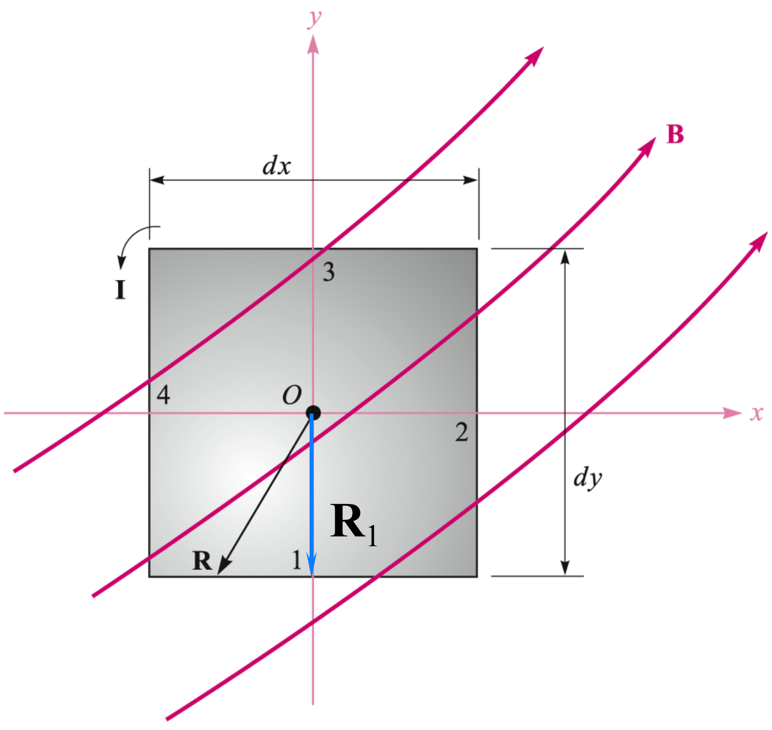
$$d\mathbf{F}_1 = I\,dx(B_{0y}\mathbf{a}_z - B_{0z}\mathbf{a}_y)$$

For side 1, the lever arm extends from the origin to the midpoint of the side, and is given by:

$$\mathbf{R}_1 = -\tfrac{1}{2}dy\,\mathbf{a}_y$$

The differential torque acting on side 1 is then:

$$\begin{aligned} d\mathbf{T}_1 &= \mathbf{R}_1 \times d\mathbf{F}_1 \\ &= -\tfrac{1}{2}dy\,\mathbf{a}_y \times I\,dx(B_{0y}\mathbf{a}_z - B_{0z}\mathbf{a}_y) \\ &= -\tfrac{1}{2}dx\,dy\,I\,B_{0y}\mathbf{a}_x \end{aligned}$$



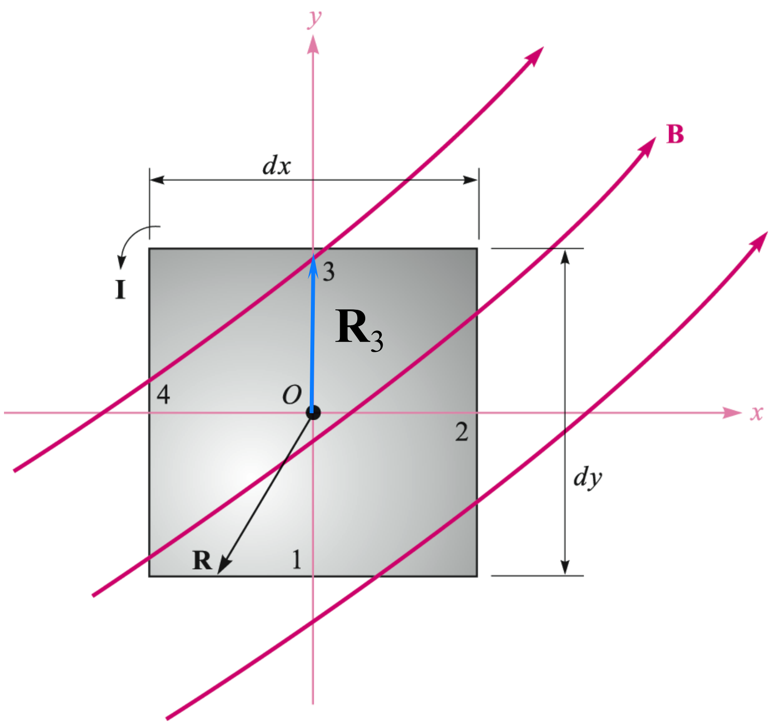
Torque on a Differential Current Loop

We next consider the opposite side (3), and using similar reasoning, find the differential torque acting on side 3:

$$d\mathbf{T}_3 = \mathbf{R}_3 \times d\mathbf{F}_3$$

$$= \tfrac{1}{2} dy\, \mathbf{a}_y \times (-I\, dx\, \mathbf{a}_x \times \mathbf{B}_0)$$

$$= -\tfrac{1}{2} dx\, dy\, IB_{0y}\mathbf{a}_x = d\mathbf{T}_1 \text{ (!)}$$



The total differential torque acting on sides 1 and 3 is then:

$$d\mathbf{T}_1 + d\mathbf{T}_3 = -dx\, dy\, IB_{0y}\mathbf{a}_x$$

Then, using the same reasoning, the total differential torque acting on sides 2 and 4 is:

$$d\mathbf{T}_2 + d\mathbf{T}_4 = dx\, dy\, IB_{0x}\mathbf{a}_y$$

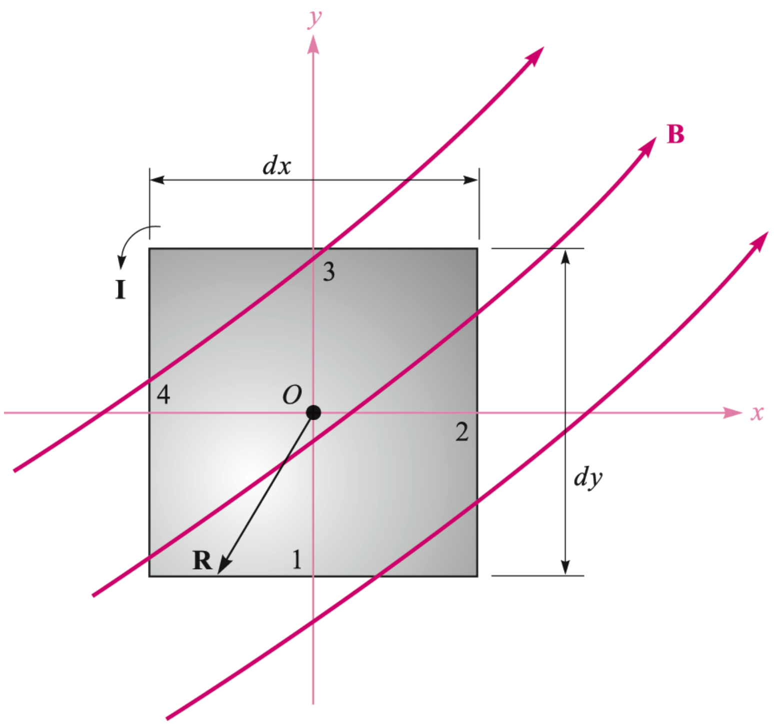
Torque on a Differential Current Loop

Now, with:

$$d\mathbf{T}_1 + d\mathbf{T}_3 = -dx\, dy\, IB_{0y}\mathbf{a}_x$$

and

$$d\mathbf{T}_2 + d\mathbf{T}_4 = dx\, dy\, IB_{0x}\mathbf{a}_y$$



The total torque on all four sides is:

$$d\mathbf{T} = I\, dx\, dy(B_{0x}\mathbf{a}_y - B_{0y}\mathbf{a}_x)$$

The terms in parenthesis can be written as a cross product:

$$d\mathbf{T} = I\, dx\, dy(\mathbf{a}_z \times \mathbf{B}_0)$$

Finally resulting in:

$$d\mathbf{T} = I\, d\mathbf{S} \times \mathbf{B}$$

where the differential loop area vector is defined using the right hand convention:

$$d\mathbf{S} = dxdy\, \mathbf{a}_z$$

Fingers in direction of current, thumb in direction of **S**

$d\mathbf{S}$ points out of the screen in this example

Differential Magnetic Moment

Having found the torque on a differential current loop:

$$d\mathbf{T} = I\, d\mathbf{S} \times \mathbf{B}$$

Define the *differential magnetic dipole moment* (*magnetic strength*) as the product of the current and the differential area vector:

$$d\mathbf{m} = I\, d\mathbf{S}$$

from which:

$$d\mathbf{T} = d\mathbf{m} \times \mathbf{B}$$

Torque on a Large-Scale Current Loop

If we remove the restriction on differential size, and assume *uniform magnetic flux density* over the entire loop area, the differential result

$$d\mathbf{T} = d\mathbf{m} \times \mathbf{B}$$

becomes:

$$\mathbf{T} = I\mathbf{S} \times \mathbf{B} = \mathbf{m} \times \mathbf{B}$$

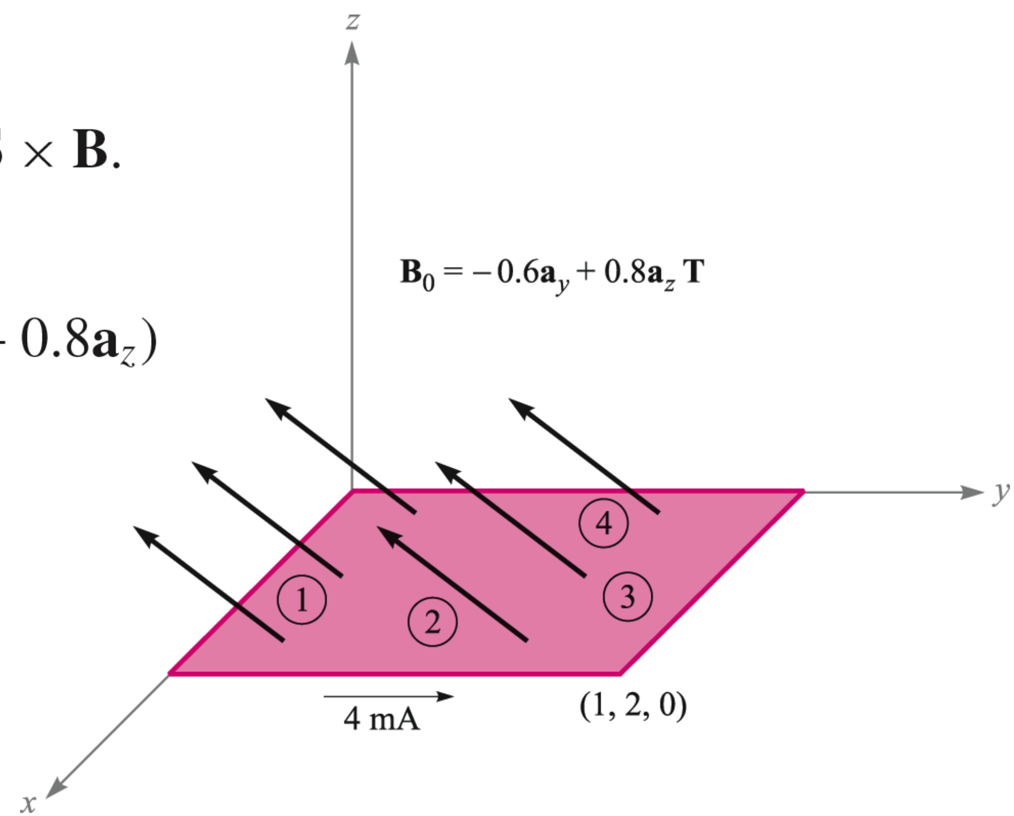
These results are independent of the shape of the loop -- the area and orientation are all that matter.

Example

Calculate the torque by using $\mathbf{T} = I\mathbf{S} \times \mathbf{B}$.

$$\mathbf{T} = 4 \times 10^{-3}[(1)(2)\mathbf{a}_z] \times (-0.6\mathbf{a}_y + 0.8\mathbf{a}_z)$$

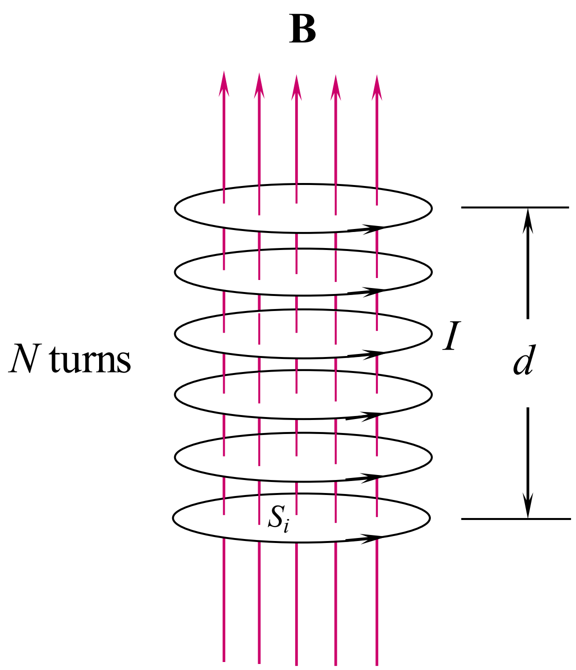
$$= 4.8\mathbf{a}_x \text{ mN} \cdot \text{m}$$



Thus, the loop tends to rotate about an axis parallel to the positive x axis. The small magnetic field produced by the 4 mA loop current tends to line up with $\mathbf{B}_0$.

Inductance Definition

Having the flux linkage (i.e. Total Flux):



$$\lambda = \sum_{i=1}^{N} \Phi_i \qquad \text{where} \qquad \Phi_i = \int_{S_i} \mathbf{B}_i \cdot d\mathbf{S}_i$$

The inductance of the device is defined as the flux linkage per unit current, or

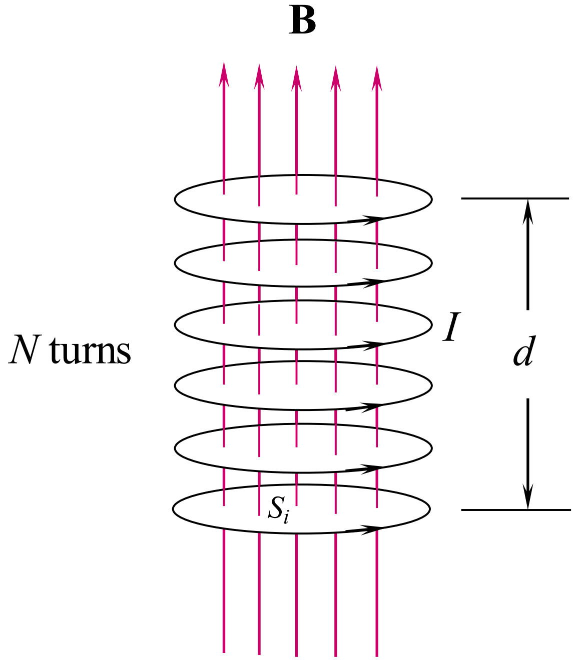
$$\boxed{L \equiv \frac{\lambda}{I} = \frac{N\Phi}{I}}$$

where the last equality applies if all turns are identical

The units of inductance are Weber-turns per Ampere, where 1 Wb-t/A is defined as one *Henry* [H].

Solenoid Inductance

For a long solenoid, having turn density n, and core permeability μ, the magnetic flux density has magnitude:



$$B = \mu n I = \frac{\mu N I}{d}$$

Then, assuming equal flux densities through N identical turns, the flux linkage is

$$\lambda = N\Phi = NBS = \frac{\mu N^2 I S}{d}$$

…and the inductance is:

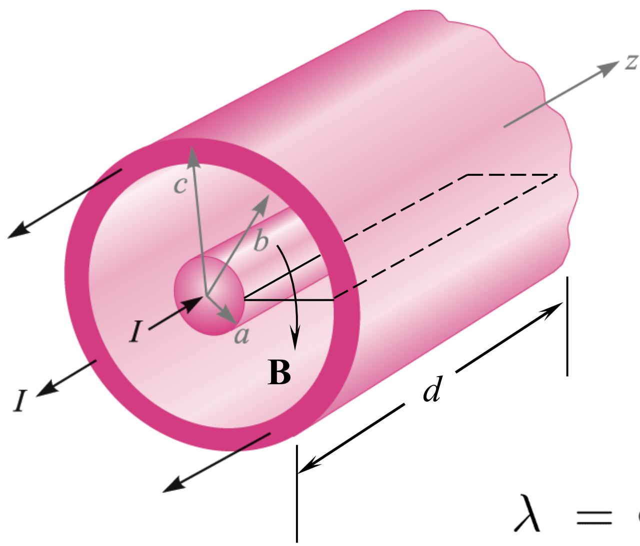
$$L \equiv \frac{\lambda}{I} = N^2 \frac{\mu S}{d}$$

It is interesting to compare this result to the capacitance of a parallel-plate capacitor, having plate area S, plate spacing, d, and dielectric permittivity, ϵ:

$$C = \frac{\epsilon S}{d}$$

Example: Inductance of a Coaxial Line

Consider a length d of coax, as shown here. The magnetic field strength between conductors is:



$$H_\phi = \frac{I}{2\pi\rho} \quad (a < \rho < b)$$

and so: $$\mathbf{B} = \mu_0 \mathbf{H} = \frac{\mu_0 I}{2\pi\rho}\mathbf{a}_\phi$$

The magnetic flux is now the integral of **B** over the flat surface between radii a and b, and of length d along z. As we have only one turn ($N = 1$), the result is also the flux linkage:

$$\lambda = \Phi = \int_S \mathbf{B}\cdot d\mathbf{S} = \int_0^d \int_a^b \frac{\mu_0 I}{2\pi\rho}\mathbf{a}_\phi \cdot d\rho\, dz\, \mathbf{a}_\phi = \frac{\mu_0 I d}{2\pi}\ln\frac{b}{a}$$

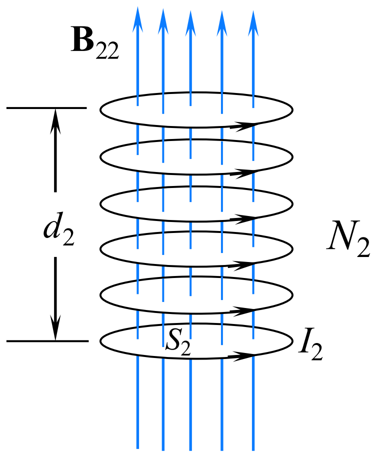
Now, with $d = 1$, the inductance per unit length is:

$$L = \frac{\lambda}{I} = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \quad \text{H/m}$$

Two Inductors

Suppose we have two solenoids, having different specifications as indicated:

Coil 2

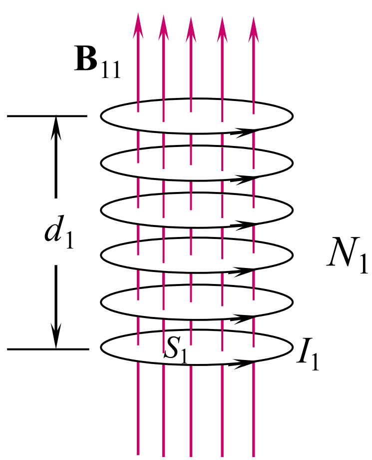


The *self linkage* and *self inductance* of each coil are determined in the manner that we used before, assuming identical fluxes through each turn.

$$\lambda_{22} = N_2 \Phi_{22} = N_2 \int_{S_2} \mathbf{B}_{22} \cdot d\mathbf{S}_2$$

and $$L_{22} = \frac{\lambda_{22}}{I_2} = N_2^2 \frac{\mu_2 S_2}{d_2}$$

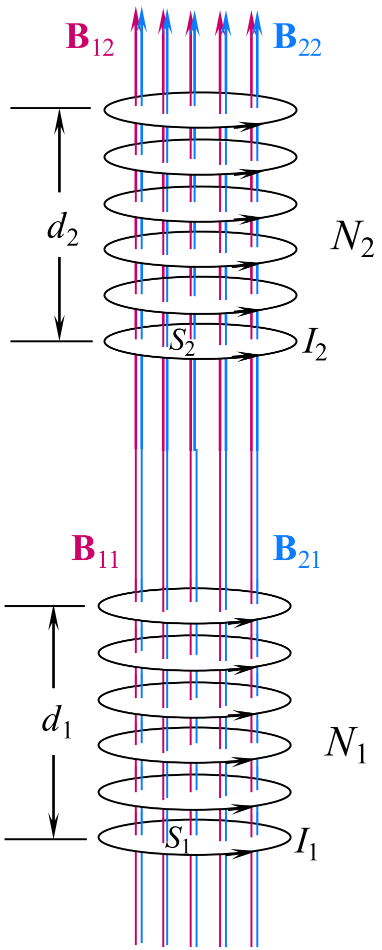
Coil 1



$$\lambda_{11} = N_1 \Phi_{11} = N_1 \int_{S_1} \mathbf{B}_{11} \cdot d\mathbf{S}_1$$

and $$L_{11} = \frac{\lambda_{11}}{I_1} = N_1^2 \frac{\mu_1 S_1}{d_1}$$

Interaction Between Inductors



Actually, the magnetic fields generated by each coil will link the other, as shown here. This flux overlap is the basis of *mutual inductance*.

Throughout this discussion, the field in red is that generated by Coil 1, while the blue field is generated by Coil 2

With both currents on, all the fields indicated here will be present. The fields and other quantities are kept track of by the subscripts, the meaning of which is:

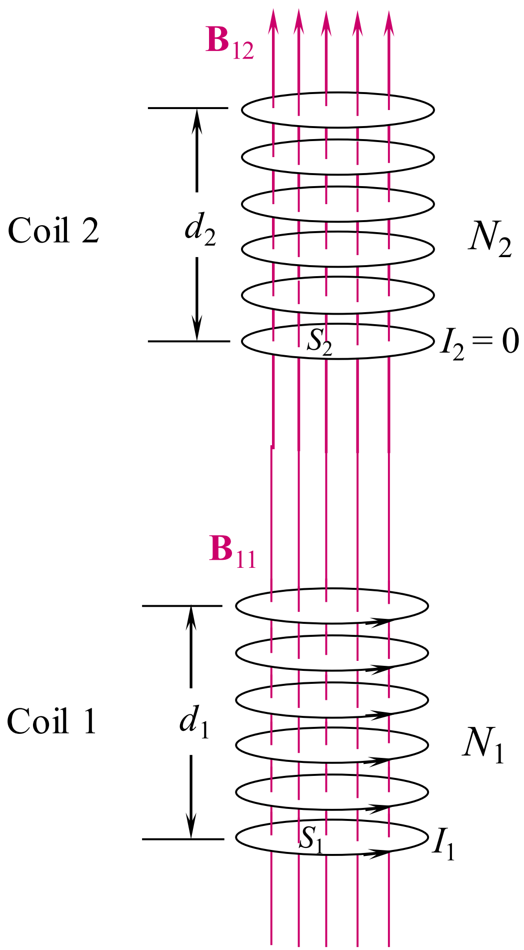
$$\mathbf{B}_{ij}$$

arising from coil i — evaluated within coil j

$$i, j = 1, 2$$

Note that the diagrams shown here are oversimplified, because there will be significant spreading of the crossover fields, $\mathbf{B}_{12}$ and $\mathbf{B}_{21}$.

Mutual Inductance, M_{12}



In this case, current in Coil 2 is turned off, leaving only the flux density generated by Coil 1, $\mathbf{B}_{12}$, existing within Coil 2.

The *mutual linkage* between Coils 1 and 2 is found through:

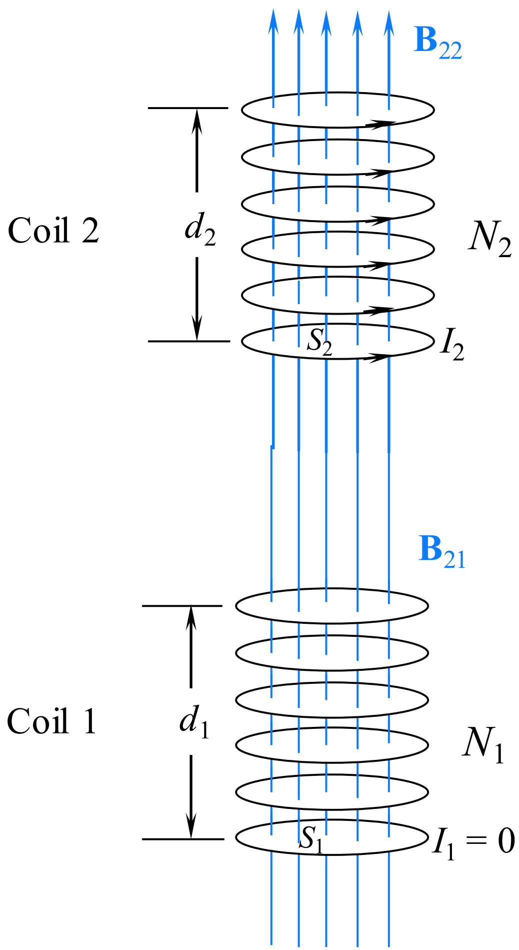
$$\lambda_{12} = N_2 \Phi_{12} = N_2 \int_{S_2} \mathbf{B}_{12} \cdot d\mathbf{S}_2$$

… and the *mutual inductance* between Coils 1 and 2 is defined as:

$$M_{12} \equiv \frac{\lambda_{12}}{I_1}$$

Again, we oversimplify here, in that the non-uniformity of $\mathbf{B}_{12}$ may likely require a turn-by-turn evaluation of the flux in Coil 2, in order to obtain the mutual linkage (in the worst case).

Mutual Inductance, M_{21}



In this case, current in Coil 1 is turned off, leaving only the flux density generated by Coil 2, $\mathbf{B}_{21}$, existing within Coil 1.

The *mutual linkage* between Coils 2 and 1 is found through:

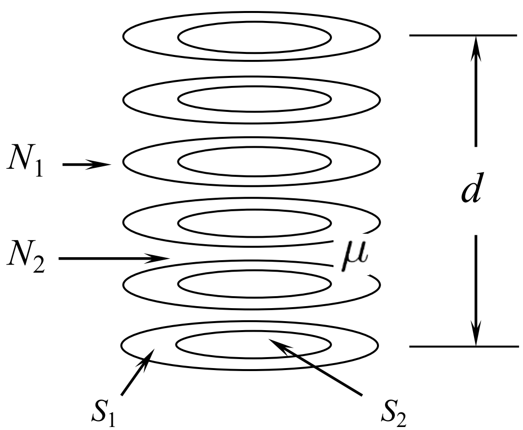
$$\lambda_{21} = N_1 \Phi_{21} = N_1 \int_{S_1} \mathbf{B}_{21} \cdot d\mathbf{S}_1$$

… and the *mutual inductance* between Coils 2 and 1 is defined as:

$$M_{21} \equiv \frac{\lambda_{21}}{I_2}$$

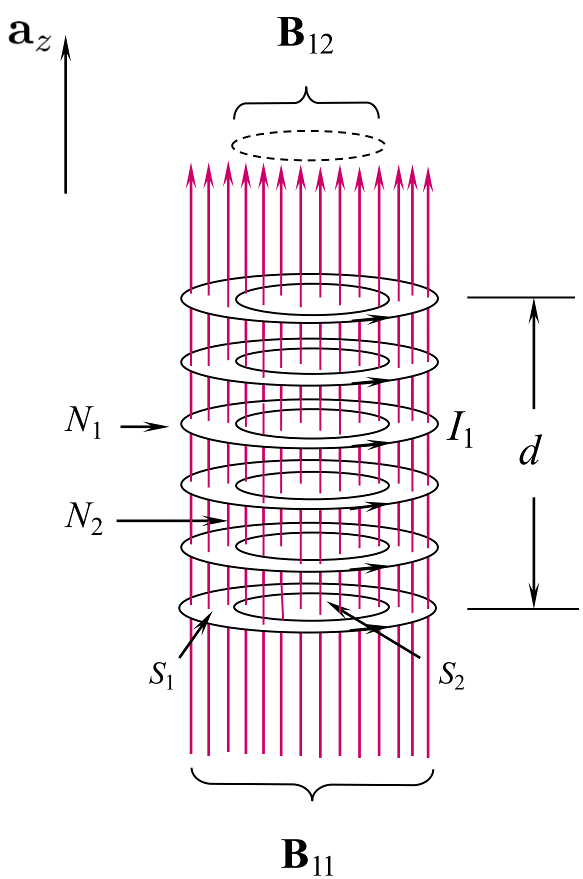
As before, the likely non-uniformity of $\mathbf{B}_{21}$ may likely require a turn-by-turn evaluation of the flux in Coil 1, in order to obtain the mutual linkage.

Example: Concentric Solenoids



In this configuration, two concentric solenoids have different numbers of turns, N_1 and N_2 (even though in the drawing the turn count would appear to be the same). Both coils have the same length, d. The area of each identical turn in the two coils is S_1 for the outer coil, and S_2 for the interior coil. The core permeability is μ

Mutual Inductance, M_{12}



With the outer coil current I_1 turned on, the interior flux exists throughout the outer coil volume, and consequently throughout the volume of the inner coil as well.

Flux density $\mathbf{B}_{12}$ resides inside coil 2 (and in coil 1 as well)

Assuming a long coil, the flux density is:

$$\mathbf{B}_{12} = \frac{\mu N_1 I_1}{d} \mathbf{a}_z$$

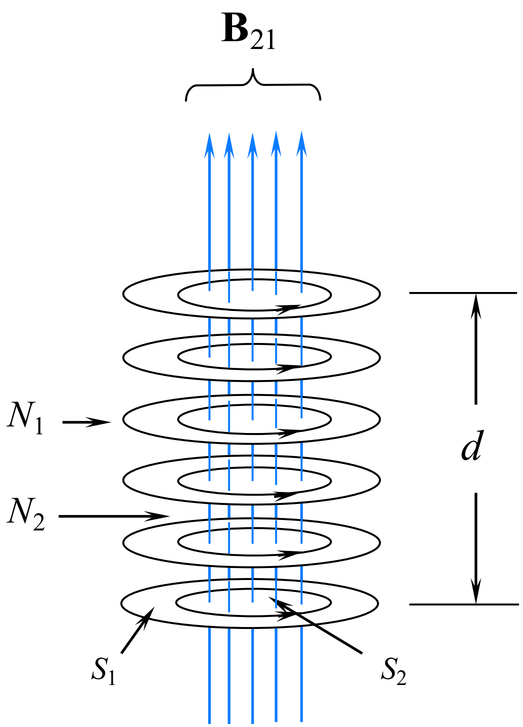
The mutual linkage between coils 1 and 2 is then:

$$\lambda_{12} = N_2 \Phi_{12} = N_2 \frac{\mu N_1 I_1}{d} S_2$$

.. and the mutual inductance is:

$$M_{12} = \frac{\lambda_{12}}{I_1} = N_1 N_2 \frac{\mu S_2}{d}$$

Mutual Inductance, M_{21}



With the inner coil current I_2 turned on, the interior flux exists throughout the inner coil volume, which overlaps with the outer coil volume

Flux density $\mathbf{B}_{21}$ resides inside coil 2 (and in coil 1 as well), but the flux in the coil 1 volume is confined within the volume of coil 2.

The coil 2 flux density (that resides in both coils) is:

$$\mathbf{B}_{21} = \frac{\mu N_2 I_2}{d}\,\mathbf{a}_z$$

The mutual flux linkage between coils 2 and 1 is then:

$$\lambda_{21} = N_1 \Phi_{21} = N_1 \frac{\mu N_2 I_2}{d} S_2$$

…and the mutual inductance is:

$$M_{21} = \frac{\lambda_{21}}{I_2} = N_1 N_2 \frac{\mu S_2}{d} = M_{12}$$

A Property of the Mutual Inductances

The foregoing example illustrates an important property of the mutual inductances between any pair of inductors:

They're equal!

$$M_{12} = M_{21}$$

Exercises

8.2. Compare the magnitudes of the electric and magnetic forces on an electron that has attained a velocity of 10^7 m/s. Assume an electric field intensity of 10^5 V/m, and a magnetic flux density associated with that of the Earth's magnetic field in temperate latitudes, 0.5 gauss. We use the Lorentz Law, $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, where $|\mathbf{B}| = 0.5 \text{ G} = 5.0 \times 10^{-5}$ T. We find

$$|\mathbf{F}_e| = (1.6 \times 10^{-19}\,\text{C})(10^5\,\text{V/m}) = \underline{1.6 \times 10^{-14}\,\text{N}}$$

$$|\mathbf{F}_m| = (1.6 \times 10^{-19}\,\text{C})(10^7\,\text{m/s})(5.0 \times 10^{-5}\,\text{T}) = \underline{8.0 \times 10^{-17}\,\text{N}} = 0.005|\mathbf{F}_e|$$

8.3. A point charge for which $Q = 2 \times 10^{-16}$ C and $m = 5 \times 10^{-26}$ kg is moving in the combined fields $\mathbf{E} = 100\mathbf{a}_x - 200\mathbf{a}_y + 300\mathbf{a}_z$ V/m and $\mathbf{B} = -3\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z$ mT. If the charge velocity at $t = 0$ is $\mathbf{v}(0) = (2\mathbf{a}_x - 3\mathbf{a}_y - 4\mathbf{a}_z) \times 10^5$ m/s:

a) give the unit vector showing the direction in which the charge is accelerating at $t = 0$: Use $\mathbf{F}(t = 0) = q[\mathbf{E} + (\mathbf{v}(0) \times \mathbf{B})]$, where

$$\mathbf{v}(0) \times \mathbf{B} = (2\mathbf{a}_x - 3\mathbf{a}_y - 4\mathbf{a}_z)10^5 \times (-3\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z)10^{-3} = 1100\mathbf{a}_x + 1400\mathbf{a}_y - 500\mathbf{a}_z$$

So the force in newtons becomes

$$\mathbf{F}(0) = (2 \times 10^{-16})[(100 + 1100)\mathbf{a}_x + (1400 - 200)\mathbf{a}_y + (300 - 500)\mathbf{a}_z] = 4 \times 10^{-14}[6\mathbf{a}_x + 6\mathbf{a}_y - \mathbf{a}_z]$$

The unit vector that gives the acceleration direction is found from the force to be

$$\mathbf{a}_F = \frac{6\mathbf{a}_x + 6\mathbf{a}_y - \mathbf{a}_z}{\sqrt{73}} = \underline{.70\mathbf{a}_x + .70\mathbf{a}_y - .12\mathbf{a}_z}$$

b) find the kinetic energy of the charge at $t = 0$:

$$\text{K.E.} = \frac{1}{2}m|\mathbf{v}(0)|^2 = \frac{1}{2}(5 \times 10^{-26}\,\text{kg})(5.39 \times 10^5\,\text{m/s})^2 = 7.25 \times 10^{-15}\,\text{J} = \underline{7.25\text{ fJ}}$$